

# Space-Time Index Modulation

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**Abstract**—In this paper, we present a new multi-antenna modulation scheme, termed as *space-time index modulation (STIM)*. In STIM, information bits are conveyed through antenna indexing in the spatial domain, slot indexing in the time domain, and  $M$ -ary modulation symbols. A time slot in a given frame can be used or unused, and the choice of the slots used for transmission conveys slot index bits. In addition, antenna index bits are conveyed in every used time slot by activating one among the available antennas.  $M$ -ary symbols are sent on the active antenna in a used time slot. We study STIM in a cyclic-prefixed single-carrier (CPSC) system in frequency-selective fading channels. It is shown that, for the same spectral efficiency and single transmit RF chain, STIM can achieve better performance compared to conventional orthogonal frequency division multiplexing (OFDM). Low-complexity iterative algorithms for the detection of large-dimensional STIM signals are also presented.

**Keywords** – *Space-time index modulation, multi-antenna systems, RF chain, single-carrier systems, OFDM, low-complexity detection.*

## I. INTRODUCTION

Multi-antenna multiple-input multiple-output (MIMO) wireless systems are known to provide increased spectral and power efficiencies. Recently, various index modulation based schemes have been developed for MIMO systems to improve the achievable rate and performance with reduced hardware complexity [1]–[6]. Spatial modulation (SM) scheme performs index modulation in the spatial domain [1]. SM activates only one transmit antenna among the available transmit antennas in any given channel use. Thus, SM reduces hardware complexity by using only one transmit radio frequency (RF) chain. The index of the active antenna used for transmission conveys information bits. SM was generalized to activate multiple transmit RF chains in a given channel use, which was termed as the generalized spatial modulation (GSM) [2], [3]. It has been shown that GSM outperforms conventional MIMO systems in terms of bit error performance for a given spectral efficiency [7].

Index modulation has been exploited in domains other than the spatial domain. Indexing subcarriers in multicarrier systems like OFDM, termed as subcarrier index modulation (SIM) [4], [5], is an example of index modulation in the frequency domain. Generalized space-frequency index modulation (GSFIM) reported in [6], [8] combines the benefits of indexing over spatial and frequency domains. In GSFIM, information bits are conveyed through antenna indexing, subcarrier indexing, and modulation symbols. However, GSFIM system

does not employ antenna indexing in every channel use. The indices of the active antennas are fixed in a given GSFIM frame. This limits the rate achievable through antenna indexing. Generalized space-time shift keying (GSTSK) reported in [9] performs indexing of space-time dispersion matrices. In GSTSK, the sum of a subset of all possible space-time dispersion matrices are chosen for transmission in a certain number of channel uses. The choice of this subset conveys information bits.

In this paper, we propose a new index modulation scheme, referred to as space-time index modulation (STIM). STIM efficiently performs indexing in spatial and time domains. In STIM, transmission is carried out in frames. Each frame consists of certain number of time slots. Not all the time slots in a frame are necessarily used for transmission (i.e., no transmission takes place in certain time slots). In fact, the choice of the combination of used slots and unused slots in a frame conveys information bits through time slot indexing. On the used time slots, modulation symbols are sent on the transmit antenna chosen based on antenna index bits. Thus, information bits are conveyed through indices of the used time slots, index of the active antenna, and modulation symbols.

STIM is well suited for use in block transmission schemes. For example, STIM can be used in cyclic-prefixed single-carrier (CPSC) scheme [10], which is a block transmission scheme suited for inter-symbol interference (ISI) channels. In this context, the following two questions arise:

- 1) how does STIM used in a CPSC scheme compare in terms of rate and performance relative to conventional OFDM in ISI channels, and
- 2) how to detect STIM signals, particularly when the dimensionality (i.e., frame size) is large.

While the first question is aimed at seeing if there are benefits vis-a-vis conventional schemes, the second question is aimed at addressing the implementation complexity issue. This paper presents answers to the above questions. First, it is found that, for the same spectral efficiency and single transmit RF chain, STIM in CPSC can outperform conventional OFDM. Second, low-complexity STIM detection algorithms which scale well for large dimensions are proposed. These results suggest that STIM can be a promising modulation scheme, and has the potential for further investigations beyond what is reported in this paper.

The rest of this paper is organized as follows. In Sec. II, we present the STIM scheme, system model, and rate

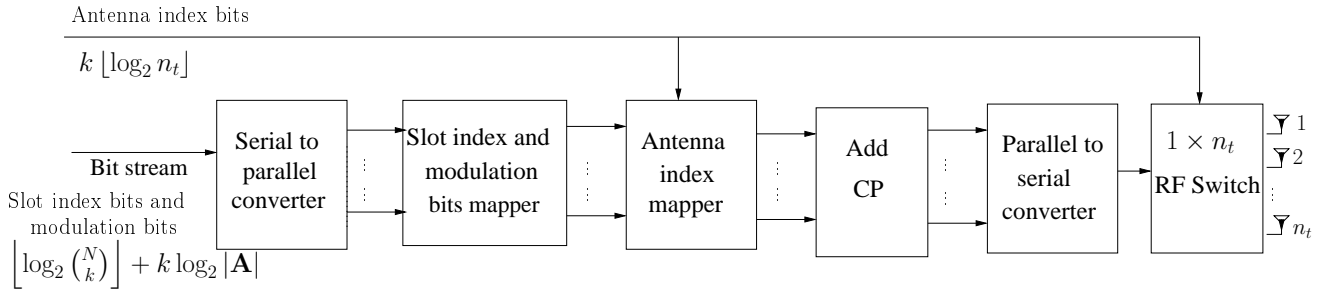


Fig. 1. STIM transmitter.

analysis. The proposed low-complexity algorithms for STIM signal detection are presented in Sec. III. Bit error performance results and discussions are presented in Sec. IV. Conclusions and scope for future work are presented in Sec. V.

## II. STIM SYSTEM MODEL

The STIM scheme has  $n_t$  transmit antennas with a single transmit RF chain (i.e.,  $n_{rf} = 1$ , where  $n_{rf}$  is the number of transmit RF chains), and  $n_r$  receive antennas. The channel is assumed to be frequency-selective with  $L$  multipaths<sup>1</sup>. Transmission is carried out in frames. Each frame consists of  $N + L - 1$  channel uses, where  $N$  denotes the length of the data part in number of channel uses and  $L - 1$  channel uses are used for transmitting cyclic prefix (CP). Information bits are conveyed through three different entities, namely, *i*) indices of the used time slots in a frame, and *ii*) index of the active transmit antenna in a channel use, and *iii*) symbols from a modulation alphabet  $\mathbb{A}$ , as follows.

### A. Time slot indexing

We have  $N$  time slots in a frame available for conveying information. Among these  $N$  slots, only  $k$  slots are selected for transmission of modulation symbols. Let  $\mathbb{A}$  denote the alphabet from which the modulation symbols are drawn. There are  $\binom{N}{k}$  possibilities of choosing which  $k$  slots are used and which  $N - k$  slots are not used. Specifically, a time slot is said to be used if any one of the  $n_t$  antennas transmits a symbol from  $\mathbb{A}$  in that slot. A slot is said to be not used, if none of the antennas transmit in that slot (i.e., all antennas remain silent, where silence can be viewed as sending a 0). The  $k$  slots used for transmission in a given frame are chosen based on  $\lfloor \log_2 \binom{N}{k} \rfloor$  information bits. Let us call a given realization of the used/unused status of  $N$  slots in a frame as a ‘slot activation pattern (SAP)’. Out of  $\binom{N}{k}$  possible SAPs, only  $2^{\lfloor \log_2 \binom{N}{k} \rfloor}$  SAPs are used for slot indexing to convey  $\lfloor \log_2 \binom{N}{k} \rfloor$  bits.

<sup>1</sup>The multipath delays can take generic delay values. As a result of Nyquist sampling at the receiver, the path delays can be reduced to integer multiples of the signaling interval [11]. The powers of the channel gains at each integer multiple of the signaling interval is given by the power-delay profile (PDP). The PDP of the frequency selective fading channel captures the statistical characteristics and dependencies between the various multipaths of the channel [11]. We consider an exponential PDP as described in Sec. II-D.

*Example:* Let  $N = 8, k = 7$ . So,  $\binom{N}{k} = 8$  and  $\lfloor \log_2 \binom{8}{7} \rfloor = 3$  slot index bits. The possible SAPs here are:

$$\left\{ \begin{bmatrix} 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 \end{bmatrix}, \right. \\ \left. \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 0 & 1 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \end{bmatrix} \right\},$$

where 1’s correspond to the location of the used time slots and 0’s correspond to the location of the unused time slots. Each used slot carries one symbol from  $\mathbb{A}$ , and nothing gets transmitted in unused slots.

### B. Antenna indexing

A transmit antenna is said to be active in a given channel use if it transmits a non-zero modulation symbol in that channel use. Similarly, a transmit antenna is said to be inactive if it does not transmit any modulation symbol in that channel use. The transmit antenna activated in a given channel use is chosen based on  $\lfloor \log_2 n_t \rfloor$  information bits. Note that in an STIM frame, in each of the  $k$  used time slots the index of the active antenna can be different. In the remaining  $N - k$  slots, all the  $n_t$  transmit antennas are inactive. Since only  $k$  out of  $N$  time slots are used in an STIM frame,  $k \lfloor \log_2 n_t \rfloor$  information bits are conveyed through antenna indices in one STIM frame.

With the addition of antenna indexing, the chosen SAP in a given frame gets mapped on to a *space-time activation matrix*  $\mathbf{A}$  of dimension  $n_t \times N$ , whose entries are zeros and ones;  $\mathbf{A}_{i,j} = 1$  means a symbol from  $\mathbb{A}$  is sent on the  $i$ th antenna in the  $j$ th slot,  $i = 1, 2, \dots, n_t$ , and  $j = 1, 2, \dots, N$ ;  $\mathbf{A}_{i,j} = 0$  means 0 is sent on the  $i$ th antenna in the  $j$ th slot (i.e.,  $j$ th slot is unused). Note that only  $k$  columns in  $\mathbf{A}$  will have a non-zero entry and the remaining  $N - k$  columns will have only zero entries.

*Example:* Let  $n_t = 2, N = 8, k = 7$ . The possible ‘antenna activation patterns (AAP)’ are given by  $\{[0, 1]^T, [1, 0]^T\}$ , where a 1 corresponds to the index of the active antenna, and a 0 corresponds to the index of the inactive antenna. Therefore,  $\lfloor \log_2 n_t \rfloor = \lfloor \log_2 2 \rfloor = 1$  bit can be conveyed through the index of the active antenna in each of the  $k = 7$  used slots.

Let  $\mathbf{B}$  denote the  $n_t \times N$  STIM signal matrix without CP. The matrix  $\mathbf{B}$  is formed by using symbols from  $\mathbb{A}$  in those

entries of the  $\mathbf{A}$  matrix where  $\mathbf{A}_{i,j} = 1$ . Now, the STIM signal matrix with CP, denoted by  $\mathbf{X}$  and of size  $n_t \times N + L - 1$ , is formed by appending  $L - 1$  CP symbols to  $\mathbf{B}$ . The columns of  $\mathbf{X}$  are transmitted in  $N + L - 1$  channel uses.

*Example:* Consider a system with  $n_t = 2, N = 8, k = 7, L = 2$ , and 4-QAM. Let  $[011010100101001111000110]$  denote a possible input information bit sequence to be transmitted by the STIM transmitter. The first  $k \lfloor \log_2 n_t \rfloor = 7$  bits choose the index of the active antennas in the  $k = 7$  used time slots. The next  $\lfloor \log_2 \binom{N}{k} \rfloor = 3$  bits choose the indices of the seven used time slots. The last  $k \log_2 |\mathbb{A}| = 14$  bits choose the seven 4-QAM symbols. For this input bit sequence, the  $\mathbf{A}$  matrix can be

$$\mathbf{A} = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix},$$

and the corresponding  $\mathbf{B}$  matrix is

$$\mathbf{B} = \begin{bmatrix} 1-j & 0 & 0 & -1-j & 0 & 1-j & 0 & 0 \\ 0 & 1+j & -1-j & 0 & 1+j & 0 & 0 & -1+j \end{bmatrix},$$

where  $j = \sqrt{-1}$ . The STIM signal matrix with CP appended is then given by

$$\mathbf{X} = \begin{bmatrix} 0 & 1-j & 0 & 0 & -1-j & 0 & 1-j & 0 & 0 \\ -1+j & 0 & 1+j & -1-j & 0 & 1+j & 0 & 0 & -1+j \end{bmatrix}.$$

The block diagram of STIM transmitter is shown in Fig. 1.

### C. Analysis of achieved rate in STIM

In this subsection, we analyze the achieved rate in STIM. We also compare the rates achieved in STIM and conventional OFDM, both using a single transmit RF chain. Based on the description of STIM in the previous subsection, the achieved rate in STIM can be written as

$$R_{\text{STIM}} = \frac{1}{N + L - 1} \left[ \underbrace{k \lfloor \log_2 n_t \rfloor}_{\text{antenna index bits}} + \underbrace{\left\lfloor \log_2 \binom{N}{k} \right\rfloor}_{\text{slot index bits}} + \underbrace{k \log_2 |\mathbb{A}|}_{\text{modulation symbol bits}} \right] \text{ bpcu.} \quad (1)$$

Note that in conventional OFDM, there are no slot and antenna index bits to contribute to the achieved rate. All the subcarriers carry modulation symbols. The achieved rate in OFDM with  $N$  subcarriers is given by

$$R_{\text{OFDM}} = \frac{N \log_2 |\mathbb{A}|}{N + L - 1} \text{ bpcu.} \quad (2)$$

From (1) and (2), we can write the percentage of rate improvement offered by STIM over OFDM as

$$\text{Rate improvement } (R_I) = \frac{R_{\text{STIM}} - R_{\text{OFDM}}}{R_{\text{OFDM}}} \times 100 \%. \quad (3)$$

1) *Maximizing  $R_{\text{STIM}}$  over  $k$ :* Define  $M \triangleq \log_2 |\mathbb{A}|$  and  $A \triangleq \lfloor \log_2 n_t \rfloor$ . From (1), the achieved rate in STIM as a function of  $k$  can be written as

$$R(k) = \frac{k(A + M) + \log_2 \binom{N}{k}}{N + L - 1}. \quad (4)$$

We need to obtain a  $k$  that maximizes  $R(k)$ . The combinatorial nature of the expression for  $R(k)$  makes it difficult to obtain the value of  $k$  that maximizes  $R(k)$  through differentiation technique. Thus, we try to obtain a range of values for  $k$  over which  $R(k)$  is maximized.

Let  $k^*$  be a value of  $k$  for which  $R(k)$  is maximum. Since  $R(k)$  is a concave function<sup>2</sup>,  $k^*$  should satisfy

$$R(k^*) - R(k^* - 1) \geq 0, \quad (5)$$

$$R(k^* + 1) - R(k^*) \leq 0. \quad (6)$$

Expanding (5), we write

$$\begin{aligned} R(k^*) - R(k^* - 1) &\geq 0 \\ \Rightarrow k^*(A + M) + \log_2 \binom{N}{k^*} \\ - (k^* - 1)(A + M) - \log_2 \binom{N}{k^* - 1} &\geq 0 \\ \Rightarrow A + M &\geq \log_2 \frac{\binom{N}{k^* - 1}}{\binom{N}{k^*}} \\ \Rightarrow A + M &\geq \log_2 \frac{k^*}{N - k^* + 1} \\ \Rightarrow 2^{A+M} &\geq \frac{k^*}{N - k^* + 1}. \end{aligned} \quad (7)$$

Define  $C \triangleq 2^{A+M}$ . From (7), an upper bound for  $k^*$  can be written as

$$k^* \leq k_u \triangleq \frac{C(N + 1)}{1 + C}. \quad (8)$$

Next, we obtain a lower bound by expanding (6) as follows:

$$\begin{aligned} R(k^* + 1) - R(k^*) &\leq 0 \\ \Rightarrow (k^* + 1)(A + M) + \log_2 \binom{N}{k^* + 1} \\ - (k^*)(A + M) - \log_2 \binom{N}{k^*} &\leq 0 \\ \Rightarrow A + M &\leq \log_2 \frac{\binom{N}{k^*}}{\binom{N}{k^* + 1}} \\ \Rightarrow A + M &\leq \log_2 \frac{k^* + 1}{N - k^*} \\ \Rightarrow C &\leq \frac{k^* + 1}{N - k^*}. \end{aligned} \quad (9)$$

Now, a lower bound for  $k^*$  can be written as

$$k^* \geq k_l \triangleq \frac{CN - 1}{1 + C}. \quad (10)$$

<sup>2</sup>It is known that  $\binom{N}{k}$  is concave w.r.t  $k$  and  $\log(\cdot)$  is a monotone function. Therefore, from (4),  $R(k)$  is a concave function w.r.t  $k$ .

From (7) and (10), we can see that any  $k$  such that  $k_l \leq k \leq k_u$  will maximize  $R(k)$ . The bounds  $k_l$  and  $k_u$  are considerably tight. This can be seen by

$$k_u - k_l = \frac{C(N+1) - CN + 1}{1+C} = 1. \quad (11)$$

Therefore, maximum rate can be achieved in STIM when we choose the value of  $k$  to be  $k_m$ , where

$$k_m = \frac{k_l + k_u}{2} = \frac{C(N+1)}{1+C} - \frac{1}{2}. \quad (12)$$

2) *Maximizing  $R_I$  over  $N$* : From (1), (2), and (3), the percentage rate improvement  $R_I$  can be written as

$$R_I = \frac{k(A+M) + \log_2 \binom{N}{k} - NM}{NM}. \quad (13)$$

Let us assume<sup>3</sup>  $k = N - 1$ . Now, (13) can be written as

$$R_I = \frac{(N-1)(A+M) - NM + \log_2 N}{NM}.$$

To maximize  $R_I$  with respect to  $N$ , we evaluate  $\frac{dR_I}{dN} = 0$ :

$$\begin{aligned} \frac{dR_I}{dN} &= \frac{A+M}{N^2M} - \frac{\log_2 N}{N^2M} + \frac{1}{N^2M \log(2)} = 0, \\ \Rightarrow \log_2 N &= A+M+1.4427, \\ \Rightarrow N &= C2^{1.4427} = n_t |\mathbb{A}| 2.7183. \end{aligned} \quad (14)$$

Therefore, the percentage of rate improvement obtained through STIM over OFDM can be maximized when the number of slots  $N = C2^{1.4427}$ .

3) *Numerical results*: In Fig. 2, we plot the variation of the achieved rate of STIM as a function of  $k$ . This figure shows the values of  $R(k)$  for varying  $k \in \{1, 2, \dots, N\}$ ,  $N = 128$ ,  $n_t = 2$  (i.e.,  $A = 1$ ),  $n_{rf} = 1$ ,  $L = 4$ , and  $|\mathbb{A}| = 2, 4, 8, 16$  (i.e.,  $M = 1, 2, 3, 4$ ). It can be seen that the STIM rate is maximized only by certain values of  $k$ . Further, the values of  $k_m$  given by (12) coincide with the values of  $k$  that maximize  $R(k)$ . For  $M = 1, 2, 3, 4$ , the values of  $k_m$  are 103, 114, 121, 125, respectively.

In Fig. 3, we plot  $R_I$  as a function of  $N$ . This figure presents the values of  $R_I$  for *i*)  $n_t = 2$  (i.e.,  $A = 1$ ), and  $|\mathbb{A}| = 2$  (i.e.,  $M = 1$ ), *ii*)  $n_t = 2$  ( $A = 1$ ), and  $|\mathbb{A}| = 4$  ( $M = 2$ ), *iii*)  $n_t = 2$  ( $A = 1$ ), and  $|\mathbb{A}| = 16$  ( $M = 4$ ), and *iv*)  $n_t = 4$  ( $A = 2$ ), and  $|\mathbb{A}| = 4$  ( $M = 2$ ). The value of  $N$  obtained through simulation, that maximizes  $R_I$ , matches well with the analytical expression given by (14). For the system parameters considered in Fig. 3,  $R_I$  is maximum at *i*)  $N = 2^{3.4427} \approx 11$ , *ii*)  $N = 2^{4.4427} \approx 22$ , *iii*)  $N = 2^{6.4427} \approx 87$ , and *iv*)  $N = 2^{5.4427} \approx 44$ .

<sup>3</sup>This assumption can be reasoned as follows: for large values of  $C$ , from (10), the value of  $k^*$  can be approximated to the nearest integer as  $k^* \approx N - 1$ . Further, this can be observed in Fig. 2, where, as  $C$  (or alternatively  $M$ ) increases, the value of  $k$  for which the maximum rate  $R_{\text{STIM}}$  is achieved increases and tends towards  $N - 1$ .

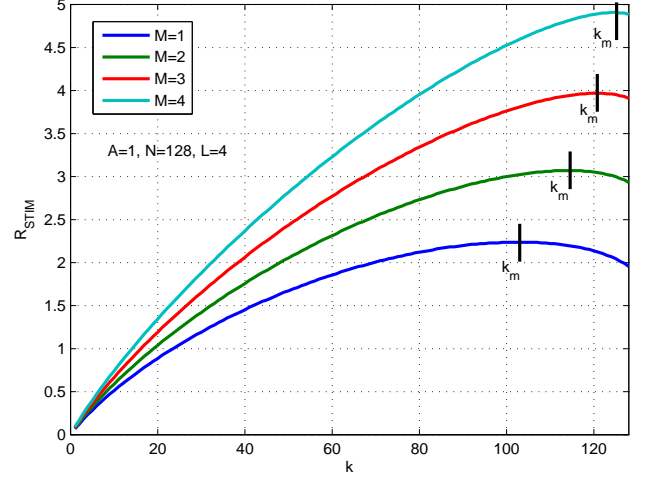


Fig. 2. Achieved rate of STIM as a function of  $k$  for  $N = 128$ ,  $n_t = 2$ ,  $n_{rf} = 1$ , and  $|\mathbb{A}| = 2, 4, 8, 16$ .

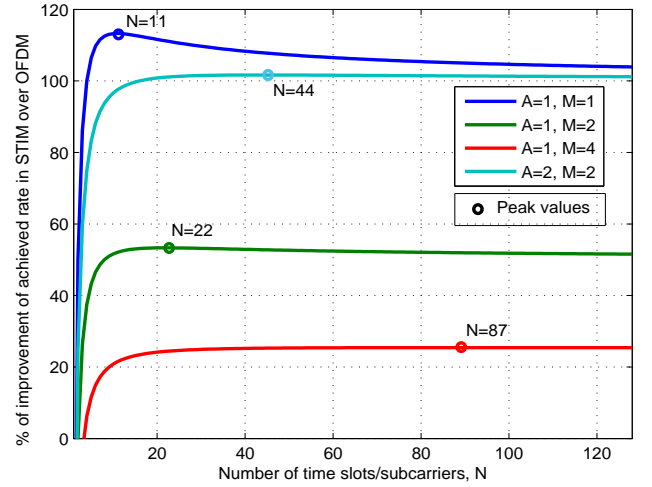


Fig. 3. Percentage rate improvement in STIM over OFDM for varying number of slots  $N$ , and varying  $n_t$  and  $\mathbb{A}$ .

#### D. Received STIM signal

Now, let us write the received STIM signal model at the receiver. Let  $h^{(j,i)}(l)$  denote the channel gain from  $i$ th transmit antenna to the  $j$ th receive antenna on the  $l$ th multipath. The power delay profile of the channel is assumed to follow exponential decaying model, i.e.,  $\mathbb{E}[|h^{(j,i)}(l)|^2] = e^{-l}$ ,  $l = 0, 1, \dots, L-1$ . Let  $\mathbf{x}_i$  denote the transmitted symbol vector of size  $n_t \times 1$  in the  $i$ th channel use,  $1 \leq i \leq N+L-1$ . Note that  $\mathbf{x}_i$  is the  $i$ th column of the matrix  $\mathbf{X}$ . We assume that the channel remains invariant in one STIM frame duration. At the receiver, after removing the cyclic prefix, the received vector can be represented as

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n}, \quad (15)$$

where  $\mathbf{n}$  is the noise vector of size  $Nn_r \times 1$  with  $\mathbf{n} \sim \mathcal{CN}(0, \sigma^2 \mathbf{I}_{Nn_r})$ ,  $\mathbf{x}$  is the vector of size  $Nn_t \times 1$  given by



$\mathbf{x} = [\mathbf{x}_L^T \ \mathbf{x}_{L+1}^T \ \dots \ \mathbf{x}_{N+L-1}^T]^T$ , and  $\mathbf{H}$  is the  $Nn_r \times Nn_t$  equivalent block circulant channel matrix given by

$$\mathbf{H} = \begin{bmatrix} \mathbf{H}_0 & 0 & 0 & 0 & \dots & 0 & \mathbf{H}_{L-1} & \dots & \mathbf{H}_1 \\ \mathbf{H}_1 & \mathbf{H}_0 & 0 & 0 & \dots & 0 & 0 & \dots & \mathbf{H}_2 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \mathbf{H}_{L-2} & \mathbf{H}_{L-3} & \mathbf{H}_{L-4} & \dots & \mathbf{H}_0 & 0 & \dots & \dots & \mathbf{H}_{L-1} \\ \mathbf{H}_{L-1} & \mathbf{H}_{L-2} & \mathbf{H}_{L-3} & \dots & \mathbf{H}_1 & \mathbf{H}_0 & 0 & \dots & 0 \\ 0 & \mathbf{H}_{L-1} & \mathbf{H}_{L-2} & \dots & \mathbf{H}_2 & \mathbf{H}_1 & \mathbf{H}_0 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & \dots & \dots & \dots & \dots & \mathbf{H}_0 \end{bmatrix},$$

where  $\mathbf{H}_l$  is the  $n_r \times n_t$  channel matrix corresponding to the  $l$ th multipath. Assuming perfect knowledge of  $\mathbf{H}$  at the receiver, the maximum likelihood (ML) detection rule is given by

$$\hat{\mathbf{x}} = \underset{\mathbf{x} \in \mathbb{S}}{\operatorname{argmin}} \|\mathbf{y} - \mathbf{H}\mathbf{x}\|^2, \quad (16)$$

where  $\mathbb{S}$  denotes the set of all possible  $\mathbf{x}$  vectors.

### III. LOW-COMPLEXITY STIM DETECTION

From (16), it can be seen that the ML detection of an STIM frame has a complexity that increases exponentially with  $N$ ,  $n_t$ . In order to enable the detection of large dimensional STIM signals, in this section, we present two low-complexity detection algorithms using message passing.

#### A. Two-stage STIM detector

In this proposed two-stage STIM detector (2SSD), detection is carried out in two stages. The first stage is a minimum mean square error (MMSE) estimator, which is followed by a message passing based detector. The purpose of the first stage detection is to provide an estimate of the indices of the active antennas in the STIM frame. Using this estimate, the message passing based detector obtains an estimate of the indices of the used time slots in the frame and the modulation symbols. Finally, these estimates are demapped to obtain an estimate of the transmitted information bit sequence.

*Stage 1* : The first stage detection is performed using MMSE estimator as

$$\hat{\mathbf{x}} = [\mathbf{H}^H \mathbf{H} + \sigma^2 \mathbf{I}]^{-1} \mathbf{H}^H \mathbf{y}, \quad (17)$$

where  $\hat{\mathbf{x}}$  is of the form  $\hat{\mathbf{x}} = [\hat{\mathbf{x}}_L^T \ \hat{\mathbf{x}}_{L+1}^T \ \dots \ \hat{\mathbf{x}}_{N+L-1}^T]^T$ . We construct the index set  $\mathcal{I} = \{I_L, I_{L+1}, \dots, I_{N+L-1}\}$ , where  $I_i$  is the index of the element with the largest magnitude in  $\hat{\mathbf{x}}_i$ , and  $L \leq i \leq N + L - 1$ . The set  $\mathcal{I}$  gives the estimates of the indices of the active antennas in the  $N$  channel uses of an STIM frame. Now, we can write (15) in the form

$$\mathbf{y} = \bar{\mathbf{H}}\mathbf{z} + \mathbf{n}, \quad (18)$$

where  $\mathbf{z}$  is an  $N \times 1$  vector whose elements correspond to the elements in  $\mathbf{x}$  at the indices given by the set  $\mathcal{I}$ , and  $\bar{\mathbf{H}}$  is the  $Nn_r \times N$  channel matrix obtained by choosing the columns of  $\mathbf{H}$  at the indices given by the set  $\mathcal{I}$ .

*Stage 2*: The second stage of the 2SSD algorithm is a message passing based algorithm that works on the model given

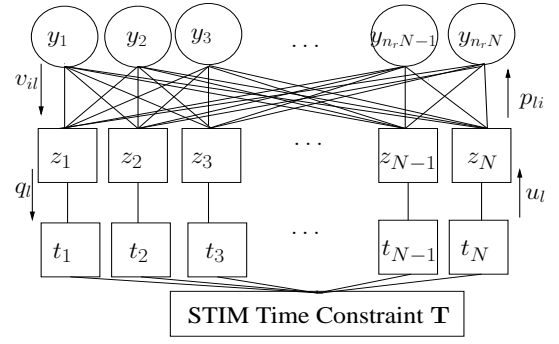


Fig. 4. The graphical model and messages in the second stage messaging passing algorithm of 2SSD.

by (18). The message passing algorithm gives the estimates of the indices of the used time slots and the modulation symbols.

Let  $t_i$  be the time slot activity indicator variable for the  $i$ th time slot. That is,  $t_i = 1$  whenever  $i$ th time slot is used, and  $t_i = 0$  otherwise. The  $i$ th time slot corresponds to the  $i$ th entry of vector  $\mathbf{z}$ . Note that  $\sum_{i=1}^N t_i = k$ . This is referred to as the STIM time constraint  $\mathbf{T}$ . Let  $\mathbf{t} \triangleq [t_1, t_2, \dots, t_N]$ . We obtain the a posteriori probabilities (APP) of the elements of  $\mathbf{t}$  and  $\mathbf{z}$ , using which the time slot index bits and the modulation symbol bits are estimated.

Figure 4 shows the graphical model for message passing. The graph consists of four sets of nodes. Messages are exchanged between these nodes in two layers: one corresponding to the probabilities of the modulation symbols and the other corresponding to the probabilities of the time slot activity indicators. For the system described in (18), the a posteriori probability is given by

$$\begin{aligned} \Pr(\mathbf{z}|\mathbf{y}) &= \Pr(\mathbf{z}, \mathbf{t}|\mathbf{y}) \\ &\propto \Pr(\mathbf{y}|\mathbf{z}, \mathbf{t})\Pr(\mathbf{z}, \mathbf{t}) \\ &= \Pr(\mathbf{y}|\mathbf{z})\Pr(\mathbf{z}|\mathbf{t})\Pr(\mathbf{t}) \\ &= \left\{ \prod_{j=1}^{n_r N} \Pr(y_j|\mathbf{z}) \prod_{i=1}^N \Pr(z_i|t_i) \right\} \Pr(\mathbf{t}). \end{aligned} \quad (19)$$

The graph is constructed so as to marginalize the above probability distribution. The following four sets of nodes are defined: *i*)  $n_r N$  observation nodes corresponding to the elements of  $\mathbf{y}$ , *ii*)  $N$  variable nodes corresponding to the elements of  $\mathbf{z}$ , *iii*)  $N$  time slot activity nodes corresponding to the elements of  $\mathbf{t}$ , and *iv*) a constraint node corresponding to the time constraint  $\mathbf{T}$ . The messages that are exchanged in layer 1 between the observation and the variable nodes produce approximate APPs of the individual elements of  $\mathbf{z}$ . The messages that are exchanged in layer 2 between the time slot activity nodes and the STIM time constraint node generate the APPs of the elements of  $\mathbf{t}$ . To generate these messages, we employ a Gaussian approximation of interference as follows.

We write the  $i$ th element of  $\mathbf{y}$  as

$$y_i = \bar{H}_{i,l}z_l + \underbrace{\sum_{j=1, j \neq l}^N \bar{H}_{i,j}z_j}_{\triangleq d_{i,l}} + n_i, \quad (20)$$

where  $i = 1, 2, \dots, n_r N$  and  $l = 1, 2, \dots, N$ . We approximate  $d_{i,l}$  to be Gaussian with mean  $\mu_{i,l}$  and variance  $\sigma_{i,l}^2$ , which are calculated as

$$\mu_{i,l} = \mathbb{E} \left[ \sum_{\substack{j=1, \\ j \neq l}}^N \bar{H}_{i,j}z_j + n_i \right], \quad (21)$$

and

$$\sigma_{i,l}^2 = \text{Var} \left( \sum_{\substack{j=1, \\ j \neq l}}^N \bar{H}_{i,j}z_j \right) + \sigma^2. \quad (22)$$

The messages passed between the nodes are given below.

*Layer 1:* The message  $v_{il}$  computed at the observation node is

$$\begin{aligned} v_{il}(z) &\triangleq \Pr(z_l = z | y_i) \\ &\approx \frac{1}{\sigma_{i,l} \sqrt{2\pi}} \exp \left( -\frac{(y_i - \mu_{i,l} - z \bar{H}_{i,l})^2}{2\sigma_{i,l}^2} \right), \end{aligned} \quad (23)$$

where  $z \in \mathbb{A} \cup 0$ . The APP of the individual elements of  $\mathbf{z}$  is obtained at the variable nodes as

$$\begin{aligned} p_{li}(z) &\triangleq \Pr(z_l = z | \mathbf{y}_{\setminus i}) \\ &\approx \prod_{j=1, j \neq i}^{n_r N} \Pr(z_l = z | y_j) \propto u_l(z^\odot) \prod_{j=1, j \neq i}^{n_r N} v_{jl}(z), \end{aligned} \quad (24)$$

where  $\mathbf{y}_{\setminus i}$  denotes the set of all elements of  $\mathbf{y}$  except  $y_i$ , and  $z^\odot = 0$  if  $z = 0$ , and  $z^\odot = 1$  if  $z \neq 0$ .

*Layer 2:* The APP estimate of  $t_l$  from the variable nodes are computed as

$$\begin{aligned} q_l(b) &\triangleq \Pr(t_l = b | \mathbf{z}) \\ &\propto \begin{cases} \sum_{z \in \mathbb{A}} \prod_{j=1}^{n_r N} v_{jl}(z), & \text{if } b = 1 \\ \prod_{j=1}^{n_r N} v_{jl}(0) & \text{if } b = 0. \end{cases} \end{aligned} \quad (25)$$

The APP estimate of  $t_l$  computed at the time slot activity nodes after processing the STIM time constraint  $\mathbf{T}$  is

$$\begin{aligned} u_l(b) &= \Pr(t_l = b | \mathbf{z}_{\setminus l}) \\ &\propto \begin{cases} \Pr(\sum_{j=1, j \neq l}^N t_j = k-1 | \mathbf{t}_{\setminus l}), & \text{if } b = 1 \\ \Pr(\sum_{j=1, j \neq l}^N t_j = k | \mathbf{t}_{\setminus l}), & \text{if } b = 0 \end{cases} \\ &\approx \begin{cases} \phi_l(k-1) & \text{if } b = 1 \\ \phi_l(k) & \text{if } b = 0, \end{cases} \end{aligned} \quad (26)$$

where  $\Pr(\sum_{j=1, j \neq l}^N t_j = k-1 | \mathbf{t}_{\setminus l})$  denotes the probability that the time slot activity pattern satisfies the STIM time constraint  $\mathbf{T}$  given that the  $l$ th time slot is used, and  $\Pr(\sum_{j=1, j \neq l}^N t_j = k | \mathbf{t}_{\setminus l})$  denotes the probability that the time slot activity pattern satisfies  $\mathbf{T}$  given that the  $l$ th time slot is unused. This

probability is evaluated as  $\phi_l = \otimes_{j=1, j \neq l}^N q_j$ , where  $\otimes$  is the convolution operator, and  $\phi_l(\cdot)$  is a probability mass function with probability masses at  $N$  points  $(0, 1, \dots, N-1)$ . The messaging passing algorithm is summarized as follows.

1. Initialize  $p_{li}(z) = \frac{1}{|\mathbb{A}_0|}$ ,  $q_l(1) = \frac{k}{N}$ ,  $q_l(0) = 1 - \frac{k}{N}$ ,  $\forall z, i, l$ .
2. Compute  $v_{il}(z)$ ,  $\forall z, l, i$ .
3. Compute  $u_l(b)$ ,  $\forall l, b$ .
4. Compute  $p_{li}(z)$ ,  $\forall l, i, z$ .
5. Compute  $q_l(b)$ ,  $\forall l, b$ .

The above steps are repeated for a fixed number of iterations. Damping of the messages  $p_{li}$ 's and  $q_l$ 's is done using a damping factor  $\Delta \in (0, 1]$  in each iteration [12]. At the end of the iterations, an estimate of the index of the used time slots are obtained by choosing the  $k$  time slots that have the largest APP. That is, a set  $\mathcal{T} = \{T_1, T_2, \dots, T_k\}$  is obtained such that  $\{q_{T_1}(1), q_{T_2}(1), \dots, q_{T_k}(1)\}$  are the  $k$  largest APP values of  $q_l(1)$ ,  $1 \leq l \leq N$ . These estimates are demapped to get the time slot index bits. Since the transmit antennas are activated only in  $k$  time slots, the  $k$  antenna indices from the set  $\mathcal{I}$  that correspond to the  $k$  used time slot estimates are chosen and demapped to obtain the antenna index bits. Finally, from  $p_{li}$ 's, the information bits corresponding to the  $k$  modulation symbols are obtained.

### B. Three-stage STIM detector

In this subsection, we present another detection algorithm called three-stage STIM detection (3SSD) algorithm, where the first two stages are the same as those in 2SSD. The third stage in the 3SSD aims to further improve the reliability of the estimated antenna index bits and the modulation symbol bits through the use of message passing over a bipartite (factor) graph. The estimates of the indices of the used time slots (i.e., the set  $\mathcal{T}$ ) obtained from the second stage are fed as input to the third stage. The third stage achieves improved performance compared to 2SSD without much increase in complexity. The message passing scheme for the third stage is described below.

Let  $\mathbb{M}$  denote the set of all possible  $n_t \times 1$  vectors that could be transmitted by the STIM transmitter in an used time slot. Therefore,  $|\mathbb{M}| = n_t |\mathbb{A}|$ . For example, for  $n_t = 2$  and BPSK, the set  $\mathbb{M}$  is given by

$$\mathbb{M} = \left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ -1 \end{bmatrix} \right\}.$$

Let  $\mathbf{w}_j \in \mathbb{M}$  be the  $n_t$ -length vector transmitted in the  $j$ th used time slot, and  $\mathbf{w} = [\mathbf{w}_1^T \ \mathbf{w}_2^T \ \dots \ \mathbf{w}_j^T \ \dots \ \mathbf{w}_k^T]^T$ . Let  $\mathbf{G}$  be an  $n_r N \times k n_t$  matrix obtained by choosing  $k n_t$  columns of the channel matrix  $\mathbf{H}$  corresponding to the estimated indices of the  $k$  used time slots given by the set  $\mathcal{T}$ . Now, the received signal can be expressed as

$$\mathbf{y} = \mathbf{G}\mathbf{w} + \mathbf{n}. \quad (27)$$

The third stage message passing works on the system model in (27), and estimates  $\mathbf{w}$ , given  $\mathbf{y}$  and  $\mathbf{G}$ . The graphical model of

this scheme consists of  $k$  variable nodes each corresponding to a  $\mathbf{w}_j$ , and  $Nn_r$  observation nodes each corresponding to a  $y_i$ . This graphical model is illustrated in Fig. 5.

The messages passed between the variable nodes and the observation nodes are constructed as follows. From (27), the received signal  $y_i$  can be written as

$$y_i = \mathbf{g}_{i,[j]} \mathbf{w}_j + \underbrace{\sum_{l=1, l \neq j}^k \mathbf{g}_{i,[l]} \mathbf{w}_l}_{\triangleq f_{i,j}} + n_i, \quad (28)$$

where  $\mathbf{g}_{i,[l]}$  is a row vector of length  $n_t$  given by  $[G_{i,(l-1)n_t+1} \ G_{i,(l-1)n_t+2} \ \cdots \ G_{i,ln_t}]$ , and  $\mathbf{w}_l \in \mathbb{M}$  is the  $n_t$ -length vector transmitted in the  $l$ th used time slot. We approximate  $f_{i,j}$  to be Gaussian with mean  $\bar{\mu}_{i,j}$  and variance  $\bar{\sigma}_{i,j}^2$ , where

$$\bar{\mu}_{i,j} = \mathbb{E} \left[ \sum_{l=1, l \neq j}^k \mathbf{g}_{i,[l]} \mathbf{w}_l + n_i \right] = \sum_{l=1, l \neq j}^k \sum_{\mathbf{s} \in \mathbb{M}} \bar{p}_{li}(\mathbf{s}) \mathbf{g}_{i,[l]} \mathbf{s}, \quad (29)$$

and

$$\begin{aligned} \bar{\sigma}_{i,j}^2 &= \text{Var} \left( \sum_{l=1, l \neq j}^k \mathbf{g}_{i,[l]} \mathbf{w}_l + n_i \right) \\ &= \sum_{l=1, l \neq j}^k \left( \sum_{\mathbf{s} \in \mathbb{M}} \bar{p}_{li}(\mathbf{s}) \mathbf{g}_{i,[l]} \mathbf{s} \mathbf{s}^H \mathbf{g}_{i,[l]}^H \right. \\ &\quad \left. - \left| \sum_{\mathbf{s} \in \mathbb{M}} \bar{p}_{li}(\mathbf{s}) \mathbf{g}_{i,[l]} \mathbf{s} \right|^2 \right) + \sigma^2, \end{aligned} \quad (30)$$

where  $\bar{p}_{ji}(\mathbf{s})$  denotes the a posteriori probability message computed at the variable nodes as

$$\bar{p}_{ji}(\mathbf{s}) \propto \prod_{m=1, m \neq i}^{Nn_r} \exp \left( \frac{-|y_m - \bar{\mu}_{m,j} - \mathbf{g}_{m,[j]} \mathbf{s}|^2}{\bar{\sigma}_{m,j}^2} \right). \quad (31)$$

The message passing schedule is as follows.

- 1) Initialize  $\bar{p}_{ji}(\mathbf{s}) = \frac{1}{|\mathbb{M}|}$ ,  $\forall j, i, \mathbf{s}$ .
- 2) Compute  $\bar{\mu}_{i,j}$ , and  $\bar{\sigma}_{i,j}^2$ ,  $\forall i, j$ .
- 3) Compute  $\bar{p}_{ji}$ ,  $\forall j, i$ .

Steps 2 and 3 are repeated for a fixed number of iterations. Damping of the messages  $\bar{p}_{ji}$ 's is done using a damping factor  $\Delta \in (0, 1]$  in each iteration. At the end of the iterations, the vector probabilities are computed as

$$\bar{p}_j(\mathbf{s}) \propto \prod_{i=1}^{Nn_r} \exp \left( \frac{-|y_i - \bar{\mu}_{i,j} - \mathbf{g}_{i,[j]} \mathbf{s}|^2}{\bar{\sigma}_{i,j}^2} \right), \quad j = 1, 2, \dots, k. \quad (32)$$

The estimates  $\hat{\mathbf{w}}_j$ s are obtained by choosing the signal vector  $\mathbf{s} \in \mathbb{M}$  that has the largest APPs. That is,

$$\hat{\mathbf{w}}_j = \underset{\mathbf{s} \in \mathbb{M}}{\text{argmax}} \bar{p}_j(\mathbf{s}). \quad (33)$$

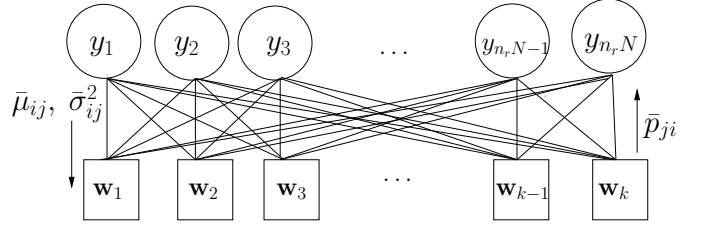


Fig. 5. The graphical model with the different messages passed in the third stage message passing algorithm of 3SSD.

The estimate of the active antenna index in the  $j$ th used time slot is obtained from  $\hat{\mathbf{w}}_j$ , which is then demapped to obtain the antenna index bits. The non-zero entries of  $\hat{\mathbf{x}}_j$  are demapped to get the modulation symbol bits. The indices in set  $\mathcal{T}$  are demapped to get the slot index bits.

#### IV. RESULTS AND DISCUSSIONS

In this section, we present the bit error rate (BER) performance of the proposed STIM detection algorithms. We compare the performance of STIM with that of OFDM. It is noted that both STIM and OFDM use a single transmit RF chain (i.e.,  $n_{rf} = 1$  for STIM and OFDM). We consider a frequency-selective channel with  $L = 2$  and exponential power delay profile presented in Sec. II-D.

In Fig. 6, we present the maximum likelihood (ML) performance of the STIM system with  $n_t = 2, N = 6, k = 5, n_r = 4$ , 4-QAM, and a spectral efficiency of 2.428 bpcu. We compare this performance with that of conventional OFDM. The OFDM systems considered for comparison are: i)  $n_t = 1, N = 6, n_r = 4$ , 8-QAM, and spectral efficiency of 2.57 bpcu, and ii)  $n_t = 1, N = 6, n_r = 4$ , 4-QAM, and spectral efficiency of 1.71 bpcu. From Fig. 6, we observe that the STIM system outperforms the conventional OFDM systems. At a BER of  $10^{-4}$  BER, STIM outperforms OFDM with 2.57 bpcu by about 6 dB, and OFDM with 1.71 bpcu by about 1.2 dB. This shows that STIM can achieve better performance than the conventional OFDM scheme as well as provide higher rate through transmit antenna and time slot indexing.

In Fig. 7, we present the BER performance of STIM detected using the proposed 2SSD and 3SSD algorithms. The value of the damping factor used is  $\Delta = 0.3$ . The STIM system considered has  $n_t = 2, N = 8, k = 7, n_r = 4$ , 4-QAM, and 2.66 bpcu spectral efficiency. For comparison purposes, we also present the performance of conventional OFDM with the same spectral efficiency of 2.66 bpcu, using the following system parameters:  $n_t = 1, N = 8, n_r = 4$ , and 8-QAM. From Fig. 7, we see that the performance of STIM with 2SSD algorithm is better than OFDM with ML detection by about 3.7 dB at a BER of  $10^{-4}$ . We also see that STIM with 3SSD algorithm outperforms OFDM with ML detection by about 4.7 dB at a BER of  $10^{-4}$ . The performance of 3SSD algorithm is better than that of the 2SSD algorithm by about 1 dB at a BER of  $10^{-4}$ . This is because the third stage message passing

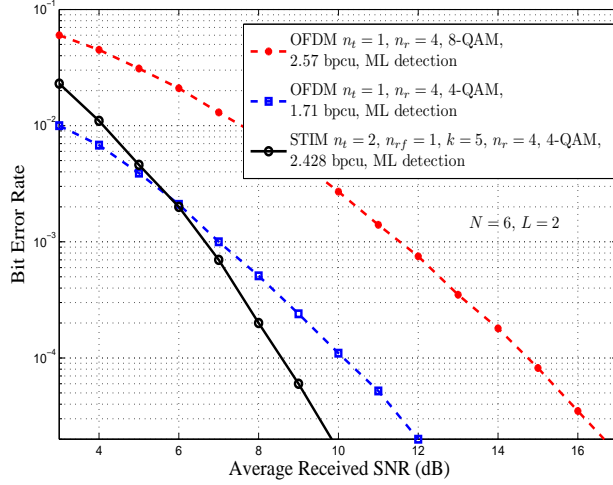


Fig. 6. BER performance of STIM and OFDM using ML detection. STIM system parameters are:  $n_t = 2$ ,  $n_{r,f} = 1$ ,  $N = 6$ ,  $k = 5$ ,  $n_r = 4$ , 4-QAM, and 2.428 bpcu. OFDM system parameters are: i)  $n_t = 1$ ,  $n_{r,f} = 1$ ,  $N = 6$ ,  $n_r = 4$ , 8-QAM and 2.57 bpcu, and ii)  $n_t = 1$ ,  $n_{r,f} = 1$ ,  $N = 6$ ,  $n_r = 4$ , 4-QAM, and 1.71 bpcu.

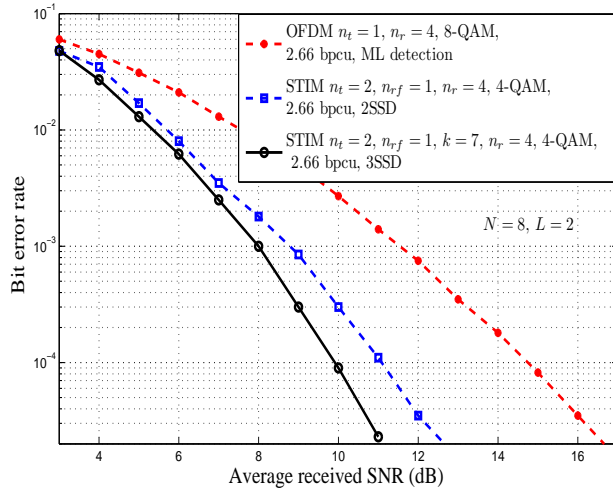


Fig. 7. Performance comparison of STIM with 2SSD and 3SSD algorithms,  $n_t = 2$ ,  $n_{r,f} = 1$ ,  $N = 8$ ,  $k = 7$ ,  $n_r = 4$ , 4-QAM, and 2.66 bpcu, with that of OFDM with ML detection,  $n_t = 1$ ,  $n_{r,f} = 1$ ,  $N = 8$ ,  $n_r = 4$ , 8-QAM, and 2.66 bpcu.

in 3SSD improves the reliability of the detected information bits. In Fig. 8, a similar performance gain in favor of STIM is observed for  $N = 12$  at 2.769 bpcu.

## V. CONCLUSION

We proposed a new modulation scheme referred to as the space-time index modulation (STIM). This modulation scheme conveys information bits through indexing of antennas and time slots, in addition to  $M$ -ary modulation symbols. This enables STIM to achieve high spectral efficiencies. We showed that, for the same spectral efficiency and single transmit RF chain, STIM can achieve better performance than conventional

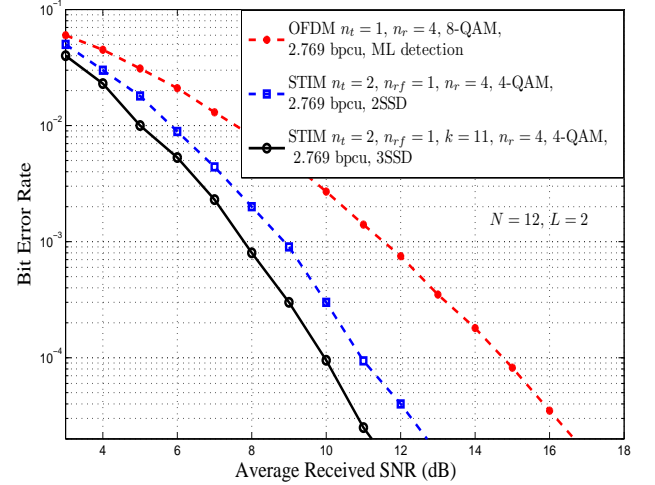


Fig. 8. Performance comparison of STIM with 2SSD and 3SSD algorithms,  $n_t = 2$ ,  $n_{r,f} = 1$ ,  $N = 12$ ,  $k = 11$ ,  $n_r = 4$ , 4-QAM, and 2.769 bpcu, with that of OFDM with ML detection,  $n_t = 1$ ,  $n_{r,f} = 1$ ,  $N = 12$ ,  $n_r = 4$ , 8-QAM, and 2.769 bpcu.

OFDM in frequency-selective fading channels. We also proposed two low-complexity detection algorithms which scale well for large dimensions. STIM with the proposed detection algorithms was shown to outperform conventional OFDM. STIM, therefore, can be a promising modulation scheme, and has the potential for further investigations beyond what is reported in this paper. For example, generalization of indexing in both space and time with more than one transmit RF chain can be investigated as future extension to this work. Also, diversity analysis of STIM and its generalized schemes is another important topic for further investigations.

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